Persistent cohomology and period reconstruction

Mikael Vejdemo-Johansson Vin de Silva Primoz Skraba

June 25, 2010

Outline

Circular coordinates

From algebraic topology, we know the isomorphism, natural in X

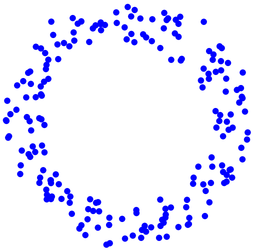
$$H^1(X;\mathbb{Z}) \cong [X, S^1]$$

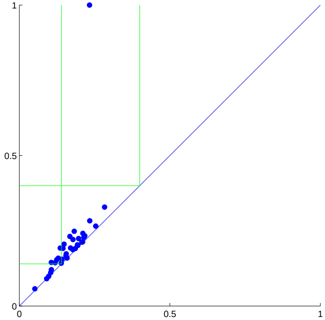
We can use this and a smoothing process to go from computed coclasses for a point cloud to circle-valued coordinate maps.

Reference

Morozov, de Silva, Vejdemo-Johansson: *Persistent Cohomology and Circular coordinates*, Proceedings of SoCG 2009; to appear: Discrete and Computational Geometry.

Examples





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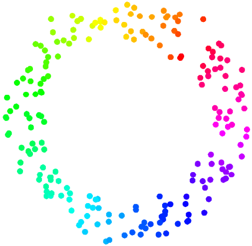

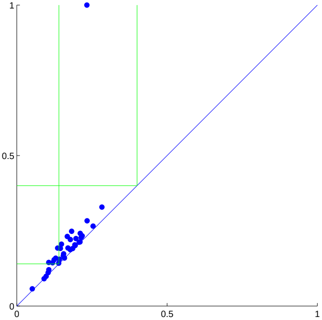

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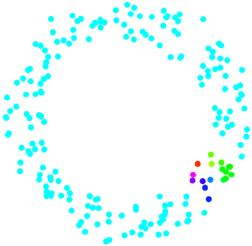

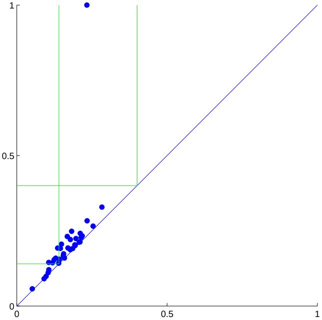

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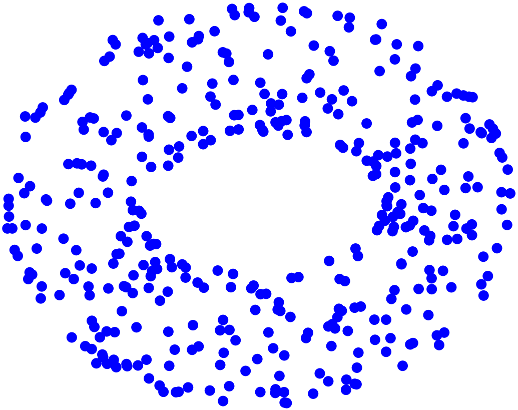

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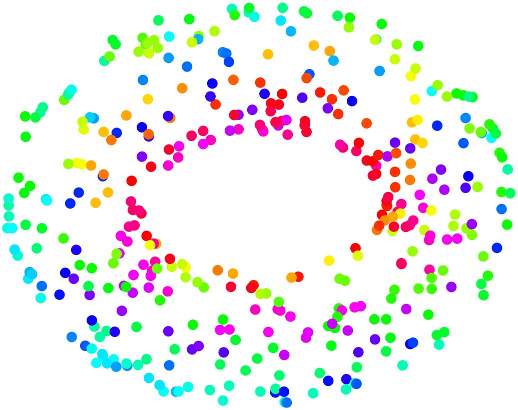

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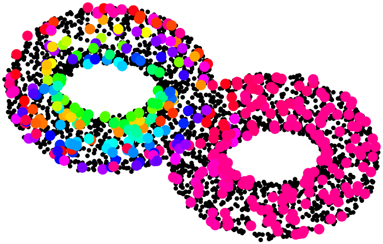

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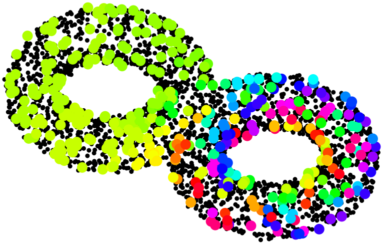

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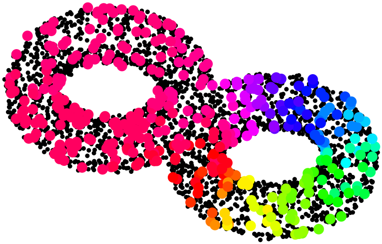

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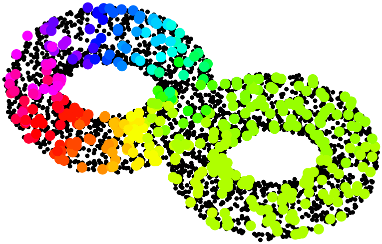


Outline

Periodic systems form circles

Completely described periodic (or even recurrent) systems form circles embedded into phase space.

Example

An oscillating system swings according to the system

$$\frac{d^2\theta}{dt^2} + a\theta = 0$$

and thus has position and velocity described by

$$\theta(t) = \frac{1}{a}\sin(at) \qquad \theta'(t) = -\cos(at)$$

describing an ellipse in the θ-θ'-plane.

Coordinatization describes recurrence

Coordinatizing a periodic (or recurrent) system yields a description of the periodicity (recurrence).

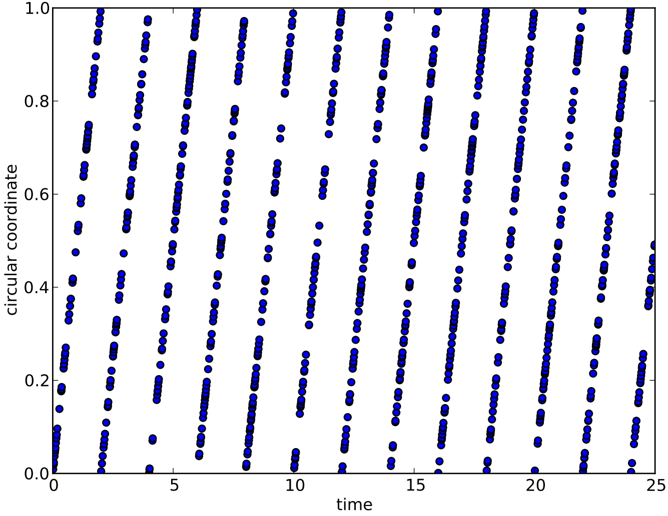
Example (cont . . .)

A circular coordinate function, recovered by sampling and a persistent cohomology computation would be

$$(\theta(t), \theta'(t)) \mapsto \frac{a}{2\pi} t \pmod{1.0}$$

The coordinate space we use is $S^1 = [0, 1]/(0 \sim 1)$.

Unrolling the coordinates



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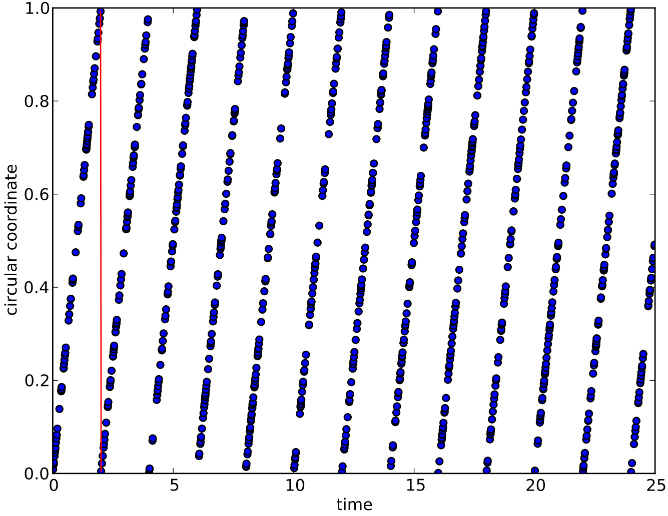

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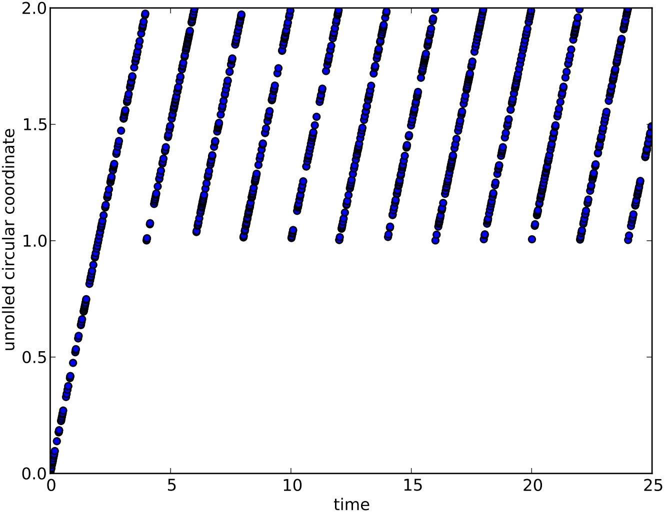

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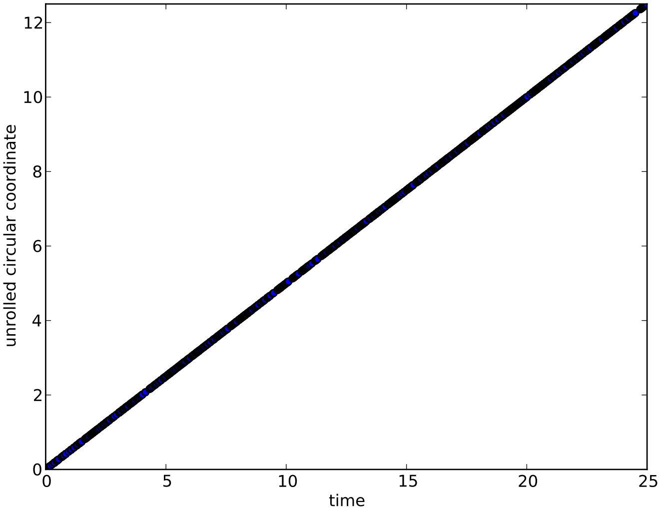

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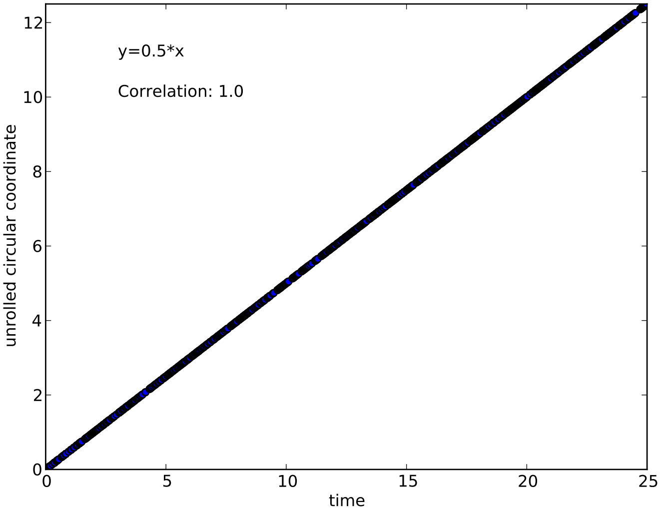


Recovering period lengths from delay embeddings

Dataset

Anderson: Mean monthly air temperature (Deg. F) Nottingham Castle 1920-1939. Source: O.D. Anderson (1976).

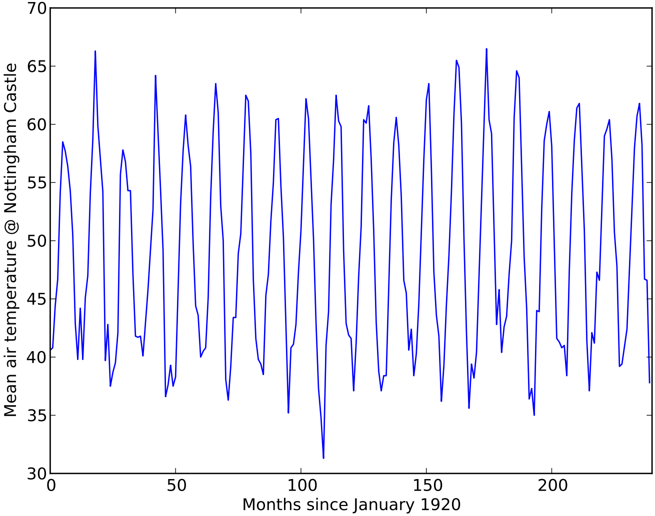
Approach

Embed $t \mapsto (a_t, a_{t+\epsilon}, a_{t+2\epsilon}) \in \mathbb{R}^3$.

Yields curve from timeseries.

If timeseries is periodic, so is the resulting embedding.

Anderson air temperature data



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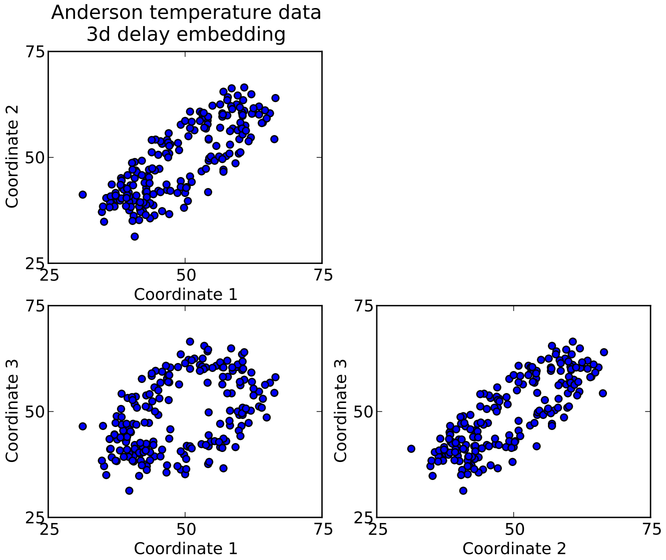

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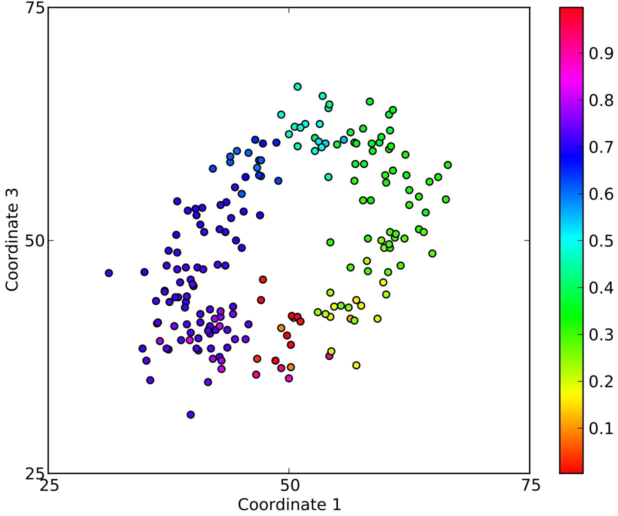

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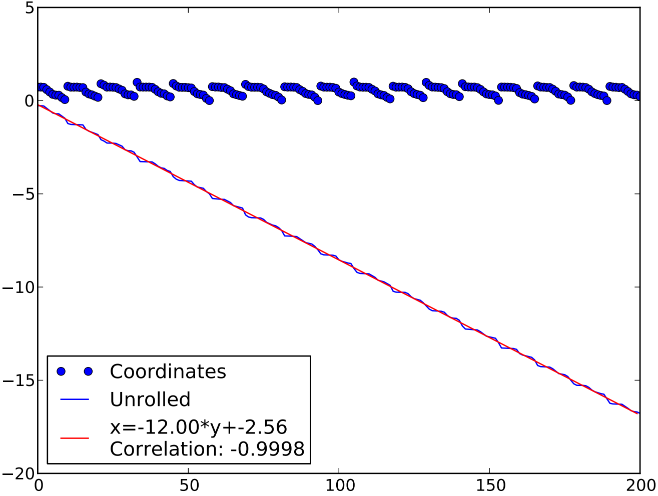

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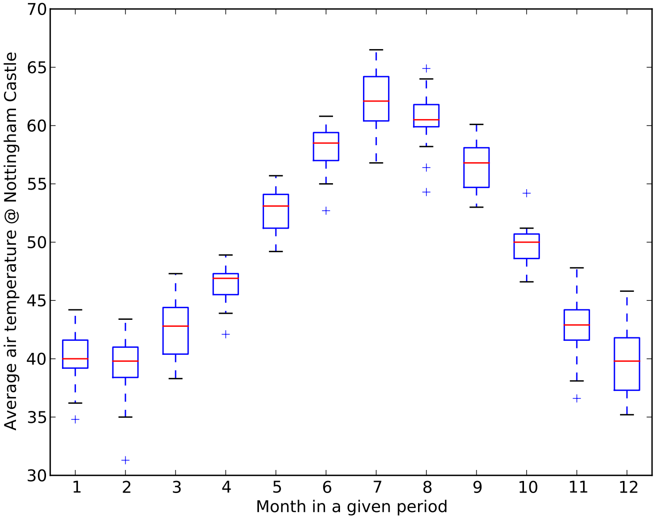

Why topological methods?

There already are methods used in dynamical systems and in signals processing for determining periodicity and periods.

We believe this system to be

- Less noise-sensitive: Fourier analysis requires actually periodic data and will easily miss periods with noise.
- Less noise-introducing: periods may be recognized by simply taking differences of points at different estimated period lengths. This is well-known to introduce extra noise.
- Multidimensional by design: phase spaces of arbitrary high dimensions are easy to deal with.

Functoriality chooses delays

Changing delay from ϵ_0 to ϵ_1 yields a map of point clouds:

$$(x_t, x_{t+\epsilon_0}, x_{t+2\epsilon_0}) \mapsto (x_t, x_{t+\epsilon_1}, x_{t+2\epsilon_1})$$

By functoriality: map in homology.

Thus we can consider persistence of features of delay embeddings over different delays; and use this to pick good delay values.

Periodic processes look like circles. Thus we look for a single dominant β_1 interval.