The Topology of Politics

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Southern Democrats

TheTopology of Politics
Obama wins
Health care reform
Outline

1. Data analysis

2. Data analysis in politics

3. Wherefore topology?
Shape of data

Fundamentally, data analysis is the task of describing the shape of data:

**Fundamental data analysis techniques**

Mean (centroid) tells us *where* the data is located.
Fundamentally, data analysis is the task of describing the shape of data:

**Fundamental data analysis techniques**

Standard deviation tells us how spread out the data is.
Shape of data

Fundamentally, data analysis is the task of describing the shape of data:

Fundamental data analysis techniques

Regression analyses fit the data to an easy to analyze model.
Shape of data

Fundamentally, data analysis is the task of describing the shape of data:

Fundamental data analysis techniques

Cluster analysis divides the data into its connected components.
Shape of data

Fundamentally, data analysis is the task of describing the shape of data:

Fundamental data analysis techniques

Principal Component Analysis (and other dimension reduction techniques) give new coordinates that more faithfully represent the data.
Outline

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Data analysis has long pedigree in political science.

- Tufte discusses improvements to techniques in a 1969 paper
- Gallups, polling, and voter prediction leverages more and stronger analysis techniques every year.

Entered into politics in the 90’s. Example of the type of analyses done: PCA of votes in House of Congress.
The shape of politics

We shall focus on one area of use: the analysis of parliamentary bodies.

Dataset

For a parliamentary session, $N$ members of parliament vote on $M$ issues. Votes form an $N \times M$-matrix $V$ of votes cast. Encoded $+1$ for Yea, $-1$ for Nay and $0$ for all other results.

c.f. Dan Rockmore’s work using similarly constructed datasets.
The shape of politics
The shape of politics
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Wherefore topology?

Topology gives qualitative information

Topology is about geometric properties not dependent on a metric: How many pieces? Are there holes? Bubbles? Can you get turned around while moving?

Topology captures continuity, connectedness, nearness

In particular, the non-dependence on a metric helps if metrics are ill-motivated, eg phylo-genetics.
A topological analysis method

In a recent PhD thesis at Stanford [Singh, ’08], a topological method for data analysis was introduced.

Fundamental topological result: Nerve lemma

Suppose a space $X$ is subdivided $X = \bigcup_i X_i$ into contractible (read simple) components. Then $X$ is equivalent to the nerve of the covering.
Wherefore topology?

The nerve of a covering
Topological application
Topological application
Topological application
Topological application
Topological application
Translate topology to statistics

Continuous function
Covering of target space
  Preimages
Connected components
Nerve complex

Measurement function on datapoints
Covering of datapoints
  Preimages
Clusters
Mapper diagram
Mapper algorithm
Mapper algorithm
Mapper algorithm
Mapper algorithm
Mapper algorithm
Mapper algorithm
Mapper algorithm

Implementation

This method is provided in a software package currently marketed by Ayasdi. Startup company founded by Gurjeet Singh (original thesis on Mapper) and Gunnar Carlsson (thesis advisor).
UK Parliament

UK Parliament

UK Parliament

UK Parliament

House of Representative.

1991
House of Representative.

1992
Wherefore topology?

House of Representative.

1993

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House of Representative.

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Wherefore topology?

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Wherefore topology?
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Wherefore topology?
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2005
Wherefore topology?

House of Representative.

2006
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2007
House of Representative.

2008
Wherefore topology?

House of Representative.

2009
House of Representative.

2010
Questions?

Future directions

- More parliaments. We have (mostly un-analyzed) data for:
  - Sweden
  - European Union
  - Canada
  - US Senate

- Network analysis: who co-sponsors with whom, what are the parliamentary cliques?

- Meme spread: who takes over turns of phrase from whom?